THE INTEGER QUANTUM HALL EFFECT

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QUANTUM HALL EXPERIMENT

K.V. Klitzing, G. Dorda, and M. Pepper PRL 45, 494 (1980)
THE BASICS

Two scales are important:

- Magnetic Field $B$
- Charge $e$
- Effective Mass $m$
- g-factor $g^*$

Cyclotron frequency

$$\omega_c = \frac{eB}{m}$$

Magnetic length

$$\ell_B = \sqrt{\frac{\hbar}{eB}}$$
THE HALL EFFECT IN A PURE SYSTEM

Landau Levels

\[ E = (n + 1/2)\hbar \omega_c \pm \frac{1}{2} g^* \mu_B B \]

Filling fraction

\[ N_e = nA, \quad N_\phi = BA/(h/e) \]
\[ \nu = \frac{N_e}{N_\phi} = \frac{nh}{eB} \]

Hall Resistance

\[ v_{\text{drift}} = E/B, \quad I = ev_{\text{drift}}w, \quad V_H = Bv_{\text{drift}}w \]
\[ R_H = \frac{V_H}{I} = \frac{B}{ne} = \frac{1}{\nu} \frac{h}{e^2} \]
LANDAU LEVELS AND DISORDER

$\rho(E)$

Localized states

Extended states

Plot (right):
Advanced Quantum Mechanics II PHYS 40202 : taken from
- http://oer.physics.manchester.ac.uk/AQM2/Notes/
Notes-4.4.htmlhttp://creativecommons.org/licenses/by-nc-sa/3.0/
PERCOLATION
Localized and extended states

\[ V(x, y) \]

Width of states

\[ \ell_B = \sqrt{\frac{\hbar}{eB}} \]

Correlation length of disorder

\[ \lambda \gg \ell_B \]
NEW PUZZLE

• If we have reduced the number of current carrying states, how is the Hall resistance not affected?
EDGE STATES

$V_{SD} \quad \mu_S - \mu_D = eV_{SD}$

Current per state

$I = \frac{ev}{L} \times \frac{L(\mu_S - \mu_D)}{hv} = \frac{e^2}{h} V_{SD}$

Difference in states on top and bottom
FLUX INSERTION

\[ V = -\frac{d\tilde{\Phi}}{dt} \]

\[ I = \sigma_{xy} V \]

Change of one flux quantum returns us to initial state

\[ \text{integer} \times e = Q = \sigma_{xy} \Delta \Phi = \sigma_{xy} \frac{h}{e} \]

\[ \sigma_{xy} = \text{integer} \times \frac{e^2}{h} \]
CHERN NUMBER

\( H = \frac{1}{2} \left[ \left( -i\hbar \partial_x + \frac{e\Theta}{L_x} \right)^2 + \left( -i\hbar \partial_y + eBx + \frac{e\Phi}{L_y} \right) \right] + V(x, y) \)

\( \Psi(x, y + L_y) = \Psi(x, y) \)

\( \Psi(x + L_x, y) = e^{iyL_x/\ell_B^2} \Psi(x, y) \)

\( \frac{j_x}{L_x} = \frac{\partial H}{\partial \Theta} \)

\[ \langle I_x \rangle = \partial_\Theta E - i\hbar [\langle \partial_\Theta \Psi | \partial_\Phi \Psi \rangle - \langle \partial_\Phi \Psi | \partial_\Theta \Psi \rangle] \partial_t \Phi \]

\[ \sigma_{xy} = -\frac{i\hbar}{(\hbar/e)^2} \int \int_0^{\hbar/e} d\Phi \, d\Theta \, \nabla \times \mathbf{v} = \frac{e^2}{h} \frac{1}{2\pi i} \oint \mathbf{v} \cdot d\mathbf{l} \]

Integer

TKNN, PRL 49, 405 (1982)
SCALING FLOW DIAGRAM

\[ g(L) = \frac{h}{e^2} \sigma_{xx}(L) L^{d-2} \]

\[ \beta(g) = \frac{\partial \ln g}{\partial \ln L} \]

Anderson insulator \hspace{1cm} (good) Metal

\[ g(L) \propto e^{-L/\xi}, \quad \beta(g) < 0 \]

\[ g(L) \propto L^{d-2}, \quad \beta(g) = d - 2 \]

\[ d > 2 \]

\[ g^* \]

\[ g \]

\[ \ln(g) \]

Insulator \hspace{1cm} Metal

\[ d = 3 \]

\[ d = 2 \]

\[ d = 1 \]
Hall conductance adds another parameter that can "flow"

Plateau transition

Quantum Hall Plateau

D. E. Khmel’nitzkii, JETP Lett. 38, 552 (1983)
CONCLUSION

• The Quantum Hall effect, due to a magnetic field, crucially depends on disorder in a system.

• The localized states don’t contribute to conductance, and the quantization can be found with either:
  • Edge states
  • Flux insertion
  • Chern numbers

• Scaling flow diagrams due of conductance provide a description of the Quantum Hall transition with disorder.